Study of Turbulence in Open Channels Using Two-Equation Models

Ahmad Sana
Sultan Qaboos University, Muscat, Sultanate of Oman
sana@squ.edu.om

Ghazi Al-Rawas
Sultan Qaboos University, Muscat, Sultanate of Oman
ghazi@squ.edu.om

Talal Etri
Sultan Qaboos University, Muscat, Sultanate of Oman
t.etri1@squ.edu.om

Abdullah Al-Mamun
Sultan Qaboos University, Muscat, Sultanate of Oman
aalmamun@squ.edu.om

Abstract

Prediction of the sediment transport in streams requires an accurate estimation of bed shear stress (for bed load) and eddy viscosity (for suspended load). In general, shallow water models employ empirical relationships to estimate the bottom shear stress. However, with the advancement of computing systems, the utilization of advanced turbulence models is getting common. In this paper, a number of model versions are reviewed based on their predictive abilities against the well-known bottom boundary layer properties in open channels and computational economy. Qualitative and quantitative comparisons have been made to infer that the choice of model versions should be based on the field application. For example, the bottom shear stress is very well predicted by the k-ε model whereas the cross-stream velocity profile and turbulent kinetic energy are predicted more efficiently by k-ε model versions. This study may be useful for researchers and practicing engineers in selecting a suitable two-equation model for calculating various bottom boundary layer properties.

Keywords: Turbulence model; Open channel; Bottom roughness; Bed shear stress; Turbulent flow; k-ε model; k-ω model

1 Introduction

The transport of sediment and pollutants in open channels essentially depend on the turbulence phenomenon among other factors. Being the most complex phenomenon to fully comprehend and estimate, turbulence computations have been the most challenging part of engineering calculations. A number of analytical and empirical models have been in vogue, prior to the widespread availability of powerful computing facilities, to estimate turbulent boundary layer properties. However, this practice changed approximately four decades ago when a number of turbulence models started to emerge and initially gained popularity among the researchers and then among the practicing engineers. Almost all the present day commercial models, used for engineering calculations for rivers and estuaries include some of these turbulence models. Nonetheless, many engineering calculations are based on empirical models or a combination of turbulence and empirical models yet. As usual, turbulence models used in research are far more complex and computationally expensive than the ones used in the field applications.

The velocity and pressure in a flowing fluid are expressed using Navier–Stokes equations. However, exact solutions of these equations are possible only for very simple flow phenomena. Most of the practical fluid flows including open channel flow are complex and classified as turbulent flows. Osborne Reynolds (1842-1912) proposed that each of the flow quantities (velocities and pressures) may be decomposed into mean and fluctuating components. Reynolds obtained what we know today as Reynolds-averaged Navier–Stokes (RANS) equations. However, even after averaging the Navier-Stokes equations, the fluctuating components remained in the RANS equations. These fluctuating components are there in the form of so-called Reynolds Stresses. Further research showed that if we try to express these Reynolds stresses in 'exact' quantities using more equations, more nonlinear terms will appear such that the number of unknowns will remain more than the available equations. This situation is termed as the '*closure problem*' which can be stated as '*total statistical description of turbulence requires an infinite set of equations*'. Later Boussinesq (1903) used the concept of eddy viscosity to define Reynolds stress in terms of the eddy viscosity and velocity gradient. Ludwig Prandtl (1875-1953) proposed the concept of mixing length to define the eddy viscosity using length and velocity scales. These developments later became the basis of the present-day turbulence models. The length and velocity scales can be represented by turbulent quantities that are expressed by turbulent transport equations.

Some studies on open channels based on Direct Numerical Simulation (DNS), Large Eddy Simulation (LES) and Reynolds Stress models have been carried out in the past. However, two-equation turbulence models have gained popularity among researchers as well as practicing engineers because of their reasonable accuracy with computational economy. Several versions of such models are reported in the literature; however, k-ε and k-ω have been the most popular two-equation models.

Jones and Launder (1972) proposed k-ε model for steady flow phenomena. However, this model has been applied to unsteady flow problems as well. Later a number of modifications were proposed in the model parameters by several researchers and some of them became popular among the engineering researchers such as Myong and Kasagi (1990) (MK), Nagano and Tagawa (1990) (NT) and Yang and Shih (1993) (YS). Sana and Tanaka (2000) reviewed some of the model versions for oscillatory boundary layers on smooth beds. Sana et al. (2007) proposed modifications in MK and NT models, which are referred in this paper as MKM and NTM, respectively. The modelling of rough surface flows has been a challenge using k-ε model. Many researchers have tried several methods, but all these methods have their shortcomings. One the other hand k-ω model, which was proposed by Wilcox (1988), can be easily used for rough as well as smooth surfaces using simple boundary conditions. it is believed that this model performs better than k-ε model near the bed. This fact motivated Menter (1994) to propose a blended k-ω model in which near the wall k-ω and far from the wall k-ε model is employed. Sana and Shuy (2002) reviewed some k-ω model versions for oscillatory boundary layers on a smooth bed and Sana et al. (2009) tested the blended k-ω model versions on a rough bed.

In the present study some popular versions of k-ε and k-ω model are used to simulate steady flow in a wide open channel having smooth or rough bed. The k-ε model versions have been used for smooth surfaces only, whereas k-ω model has been applied to all types of channel beds.

2 Theoretical Background

The turbulence models are classified based on the number of transport equations used in the model. For example, a model, which is not based on a transport equation, is termed as zero-equation model;

the ones using one and two transport equations are called one-equation and two-equation turbulence models, respectively. More advanced models like Reynolds Stress or Flux transport models utilize the transport equations for Reynolds stresses, fluxes and length scale. Another category of advanced models resolve large eddies exactly and model small eddies using a modelling assumption are called as Large Eddy Simulation models. In the present study two-equation models, namely *k-ε* and *k-ω* model versions were used for the simulation of turbulent flow in a wide-open channel (one-dimensional flow) having smooth and rough bed.

2.1 Governing Equations

The governing equations of the *k-ε* model (Eq. 1 to Eq. 4) are as follows:

$$.\frac{\partial u}{\partial t} = \frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial y} = \frac{dp}{dx} + \frac{\partial}{\partial y}\left\{(\nu + \nu_t)\frac{\partial u}{\partial y}\right\} \tag{1}$$

$$\nu_t = C_\mu f_\mu \frac{k^2}{\tilde{\varepsilon}} \tag{2}$$

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial y}\left\{\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right\} + \nu_t\left(\frac{\partial u}{\partial y}\right)^2 - \tilde{\varepsilon} - D \tag{3}$$

$$\frac{\partial \tilde{\varepsilon}}{\partial t} = \frac{\partial}{\partial y}\left\{\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \tilde{\varepsilon}}{\partial y}\right\} + C_1 f_1 \nu_t \frac{\tilde{\varepsilon}}{k}\left(\frac{\partial u}{\partial y}\right)^2 - C_2 f_2 \frac{\tilde{\varepsilon}^2}{k} + E \tag{4}$$

Where *u* is the flow velocity at y, *x* and *y* are coordinate axes, ν and ν_t are molecular and eddy viscosity, respectively, *dp*/*dx* is the pressure gradient and considered to be constant over the channel cross-section, turbulent kinetic energy, $k = \left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)/2$, turbulent kinetic energy dissipation rate, $\varepsilon = \nu \overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}}$, $\tilde{\varepsilon} = \varepsilon - D$ and *D*, *E*, $C\mu$, $f\mu$, C_1, C_2, f_1 and f_2 are model parameters.

The blended *k-ω* model equations as given by Menter (1994) are shown as Eq. (5) and Eq. (6):

$$\frac{\partial k}{\partial t} = \underbrace{\frac{\partial}{\partial y}\left\{(\nu + \nu_t \sigma_{k\omega})\frac{\partial k}{\partial y}\right\}}_{\text{Diffusion}} + \underbrace{\nu_t\left(\frac{\partial u}{\partial y}\right)^2}_{\text{Production}} - \underbrace{\beta^* \omega k}_{\text{Dissipation}} \tag{5}$$

$$\frac{\partial \omega}{\partial t} = \frac{\partial}{\partial y}\left\{(\nu + \nu_t \sigma_\omega)\frac{\partial \omega}{\partial y}\right\} + \gamma \frac{\nu + \nu_t}{\nu_t}\left(\frac{\partial u}{\partial y}\right)^2 - \beta \omega^2 + 2(1 - F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial y}\frac{\partial \omega}{\partial y} \tag{6}$$

Where, *ω* is the turbulent kinetic energy dissipation function. This blended model has been used in various types of boundary layer flows. The model parameters are given in the respective references listed in Table 1.

Table 1: Model versions used in the present study

***k-ε* model**		***k-ω* model**	
Acronym	Reference	Acronym	Reference
JL	Jones and Launder (1972)	WL	Menter (1994)
MKM	Sana et al. (2007)	BSL	Menter (1994)
NTM	Sana et al. (2007)	SST	Menter (1994)
YS	Yang and Shih (1993)		

2.2 Solution Procedure

The governing equations were converted into dimensionless form using the average velocity, U_0 distance from the bed to the axis of symmetry y_h (free-surface with no external forces) and mass density of the fluid. These dimensionless equations require only Reynolds number $Re = U_0 y_h/\nu$ and

$k_s^* = k_s/y_h$ (k_s : Nikuradse's equivalent roughness height) as input. A Crank-Nicolson type implicit finite difference scheme was used. The grid spacing was varied exponentially for better accuracy near the bottom. In cross-stream direction 100 grid points were used. Time marching scheme was used to run the simulations. The convergence was based primarily on velocity, k and ε (or ω) and then on bottom shear stress (τ_0). The convergence limit for both the steps was set to 1×10^{-6}.

3 Results and Discussion

3.1 Smooth Bed

For smooth open channels the velocity profile was predicted using k-ε model versions by JL, MKM, NTM and YS (as describe in Table 1) in Figure 1. The experimental data of two tests (Test 3: *Re* = 21,620, Test 11: *Re* = 144180) from Kirkgoz (1989) was used for comparison. It is interesting to note that JL model predicts the velocity better than the newer versions for both the tests in the logarithmic layer. However, MKM model shows better agreement in the viscous sublayer and buffer layer (Test 3).

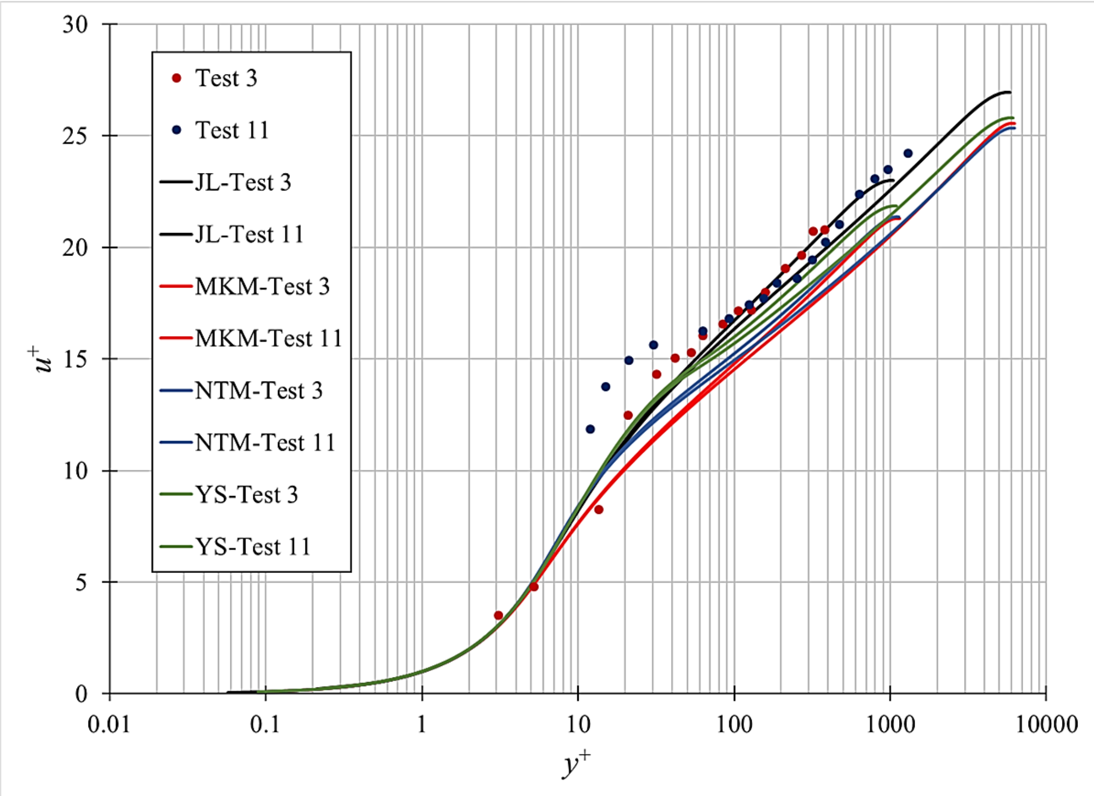

Fig. 1: Comparison of the velocity profiles between model predictions and experimental data by (Kirkgoz, 1989)

The friction factor ($f = 2\tau_0/\rho U_0^2$) is another important parameter used to calculate the bottom shear stress from the average velocity. Figure 2 shows a comparison between JL model and the relationships by Nikuradse (1933) and Colebrook (1939) who developed their relationships for pipe flow. Here these relationships are transformed for open channel using the hydraulic radius. The model shows an excellent agreement with the experimental relationships. The friction factor by Balsius (1913) and derived from logarithmic velocity profile along-with the friction factor from laminar flow ($f = 64/Re$) are also shown. The model could predict the friction factor for the whole range of flows from laminar to turbulent with precision. This characteristic of the turbulence model proves the importance of the usage of such models where it is not required to check the type of flow before applying an empirical relationship for a certain type of flow.

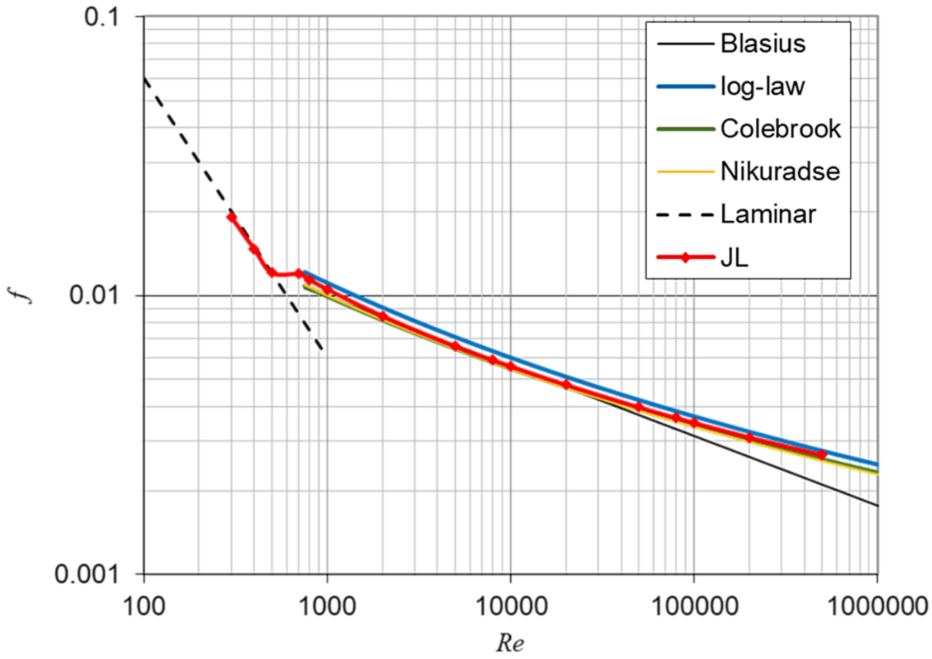

Fig. 2: Friction factor for smooth-bed open channel

3.2 Rough Bed

As mentioned before the application of *k-ε* model for flow over rough surfaces is extremely difficult. Various techniques have been used among which the wall function method is the most popular. This method employs the logarithmic relationship to specify the wall boundary condition. However, the location of the point where the boundary condition has to applied cannot be fixed because of its dependence on the roughness height. Besides, the shear stress has to be known for the application of the wall function. In other words, iterations have to be performed to find appropriate location for the boundary condition. On the other hand, the surface roughness can be easily described by a simple function of the value of *ω* at the location of the bed in case of *k-ω* model. Therefore, in the present computations only *k-ω* model is utilized for rough-bed open channels. Figure 3 shows the model parameter S_R that controls the effect of the surface roughness in *k-ω* model. Sana et al. (2009) proposed a single relationship for the whole range of roughness values, which is also shown in Figure 3.

Considering the rough surface classification, three cases were simulated, and velocity profiles and turbulence kinetic energy profiles are shown in Figure 4 and Figure 5, respectively. The velocity profiles show the fact that higher the roughness, larger is the logarithmic region. This is due to higher rate of turbulent kinetic energy transfer to the outer layer due to higher roughness. As a result, the velocity at the axis of symmetry is lesser than the case of a smaller roughness and the same Reynolds number. Moreover, the thickness of the viscous sublayer decreases with an increase in the wall roughness. All these characteristics of a rough-turbulent flow are shown efficiently by SST *k-ω* model proposed by Menter (1994). Figure 5 shows a higher production of turbulent kinetic energy for higher roughness values. Moreover, the comparison between the flow over low roughness (hydraulically smooth) and transitional roughness showing the production of turbulent kinetic energy near the wall. The model could predict all the key features of rough-bed open channel in an immaculate manner qualitatively. Further investigation using experimental or DNS data is required to judge the capabilities of the model in this regard.

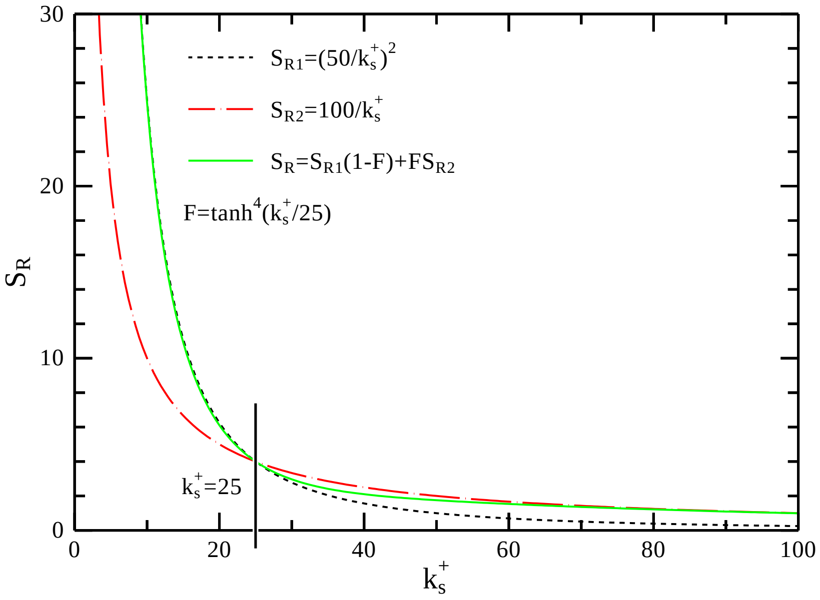

Fig. 3: Model parameter S_R in *k-ω* model to describe bed roughness

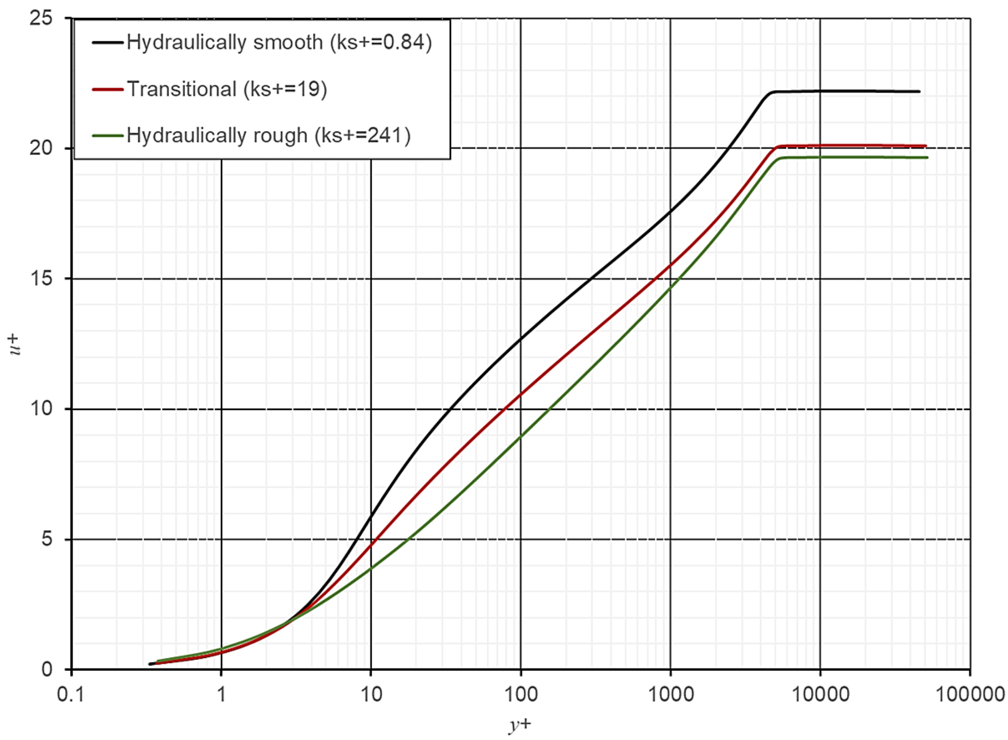

Fig. 4: Velocity profiles in a rough-bed open channel by SST *k-ω* model

3.3 Friction Factor

In case of smooth surface (laminar, transitional and turbulent flow) the friction factor is mainly a function of Reynolds number whereas for the flow over a rough surface the friction factor depends on $k_s^+(=k_s/\nu)\sqrt{\tau_0/\rho}$ (τ_0: bottom shear stress, k_s : Nikuradse's equivalent roughness height, ρ: fluid mass density). The friction factor from full-range equation derived from Tanaka and Thu (1994), Colebrook (1939) and Nikuradse (1933) are also shown in comparison with the results of SST *k-ω* model. Over the full-range of flow types, the model performs very well in predicting the friction factor.

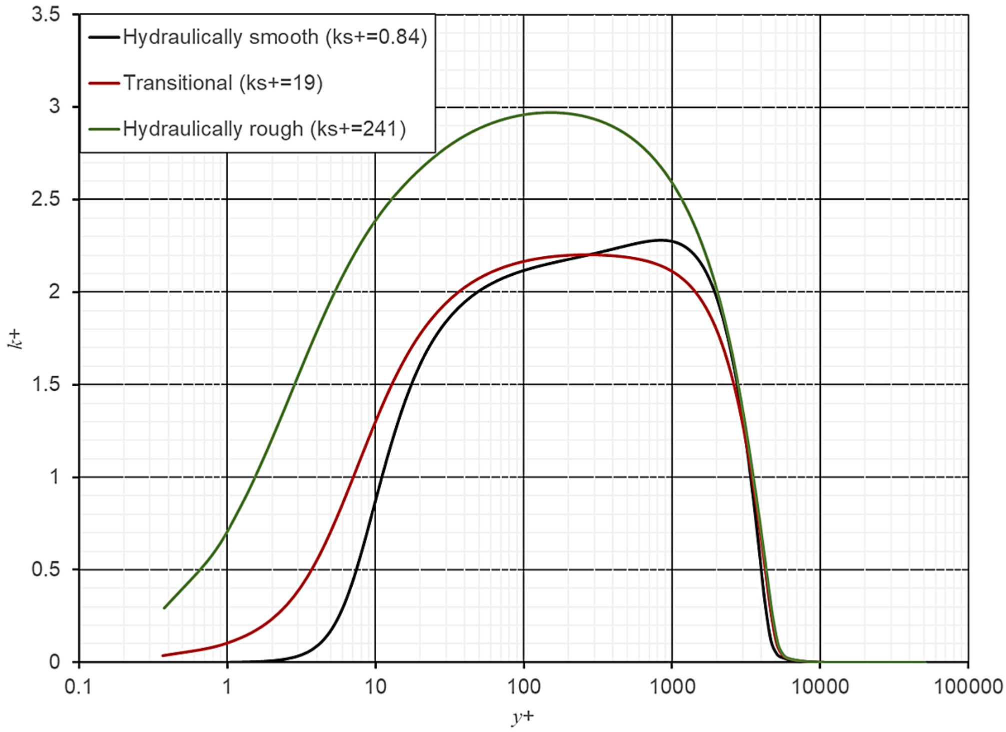

Fig. 5: Turbulent kinetic energy profiles in a rough-bed open channel by SST *k-ω* model

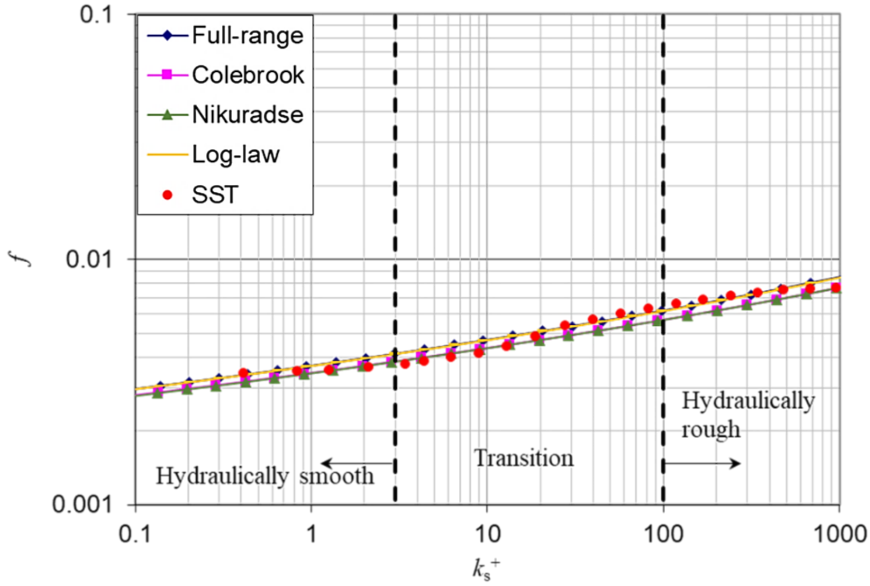

Fig. 6: Friction factor for an open channel

4 Conclusion

Two-equation turbulence models can be effectively used to analyse the whole range of laminar, transitional and turbulent flows over smooth and rough surfaces. Although it is difficult to express the roughness height in case of a *k-ε* model, the use of *k-ω* model for rough flows is rather straightforward. The original *k-ε* model (proposed by Jones and Launder, 1972) performed the best for open channel flow on a smooth surface. For rough-bed, open channel flow SST k-w model was found to be very efficient in predicting velocity, turbulent kinetic energy and friction factor. This model can be utilized for the open channel flow over the smooth as well as rough surface efficiently for the determination of fundamental flow characteristics. The information obtained from the model results can be used to estimate the sediment transport and mixing of pollutants in an open channel over a range of surfaces. This study is useful for the practicing engineers and researchers working on the boundary layer properties, sediment movement and turbulent mixing in open channels. Further investigations using experimental data is required to judge the capabilities of the turbulence models in open channel flows.

References

Blasius, H. (1913). Das Aehnlichkeitsgesetz bei Reibungsvorgängen in Flüssigkeiten. In: Mitteilungen über Forschungsarbeiten auf dem Gebiete des Ingenieurwesens. Mitteilungen über Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, vol 131. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-662-02239-9_1

Colebrook, C. (1939) Turbulent Flow in Pipes, with Particular Reference to the Transition Region between the Smooth and Rough Pipe Laws. *Journal of the Institution of Civil Engineers, 11*, 133-156. https://doi.org/10.1680/ijoti.1939.13150

Jones, W. P. & Launder, B.E., (1972). The prediction of laminarization with a two-equation model of turbulence. *International Journal of Heat and Mass Transfer 15*, 301–314.

Kirkgoz, M.S. (1989). Turbulent Velocity Profiles for Smooth and Rough Open Channel Flow, *Journal of Hydraulic Engineering*, Vol. 115, No. 11, 1543-1561

Menter, F. R. (1994). Two-equation eddy-viscosity turbulence models for engineering applications. *AIAA Journal 32*(8), 1598–1605.

Myong, H. K. & Kasagi, N. (1990). A new approach to the improvement of k–e turbulence model for wall-bounded shear flows. *JSME International Journal Series II 33*(1), 63–72.

Nagano, Y. & Tagawa, M. (1990). An improved k–e model for boundary layer flows. *Journal of Fluids Engineering—ASME 112*, 33–39.

Nikuradse, J. (1933) Stromungsgesetze in Rauhem Rohren. (Laws of Turbulent Pipe Flow in Smooth Pipes.) VDI-Forschungsheft: Germany) 361. (in German) (Translated in NACA Tech. Memo. No. 1292, 1050).

Sana A., Ghumman, A. R. & Tanaka, H. (2007). Modification of the damping function in the k–ε model to analyze oscillatory boundary layers. *Ocean Engineering, 34*(2), 320–326.

___. (2009). Modeling of a rough-wall oscillatory boundary layer using two-equation turbulence models. *J. Hydr. Eng., 135*, 60–65.

Sana, A. & Shuy, E. B. (2002). Two-equation turbulence models for smooth oscillatory boundary layers. *J. Waterway, Port, Coastal, Ocean Eng., 128*(1), 38–45.

Sana, A. & Tanaka, H. (2000). Review of k–e model to analyze oscillatory boundary layers. *Journal of Hydraulic Engineering—ASCE 126*(9), 701–710.

Tanaka, H. & Thu, A. (1994). Full-range equation of friction coefficient and phase difference in a wave-current boundary layer. *Coast. Eng. 22*, 237–254.

Wilcox, D. C. (1988). Reassessment of the scale-determining equation for advanced turbulence models. *AIAA J., 26*(11), 1299–1310.

Yang, Z. & Shih, T. H. (1993). New time scale based k2e model for near-wall turbulence. *AIAA J., 31*(7), 1191–1198.

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