Modeling Short-Term Deflection of Reinforced Concrete Slabs using the Layered Approach

Abeer A. Fageer
Department of Civil Engineering, Faculty of Engineering, University of KhartoumKhartoum, Sudan
abeerfageer97@gmail.com

Shahd K.Abdelgadir
Department of Civil Engineering, Faculty of Engineering, University of KhartoumKhartoum, Sudan
shahdkabdelgadir17@gmail.com

Anfal A. Abdelgadir
Department of Civil Engineering, Faculty of Engineering, University of KhartoumKhartoum, Sudan
Anfal.ali98.5@gmail.com

Amina M. Elbagir
Department of Civil Engineering, Faculty of Engineering, University of KhartoumKhartoum, Sudan
Aminaelbagir9@gmail.com

Mohammed A.Kabosh
Department of Civil Engineering, Faculty of Engineering, University of KhartoumKhartoum, Sudan
mohammedkabosh@hotmail.com

Amged O. Abdelatif
Department of Civil Engineering, Faculty of Engineering, University of KhartoumKhartoum, Sudan
Amged.Abdelatif@uofk.edu

Abstract

This paper examines the short-term deflection of reinforced concrete slabs taking the tension-stiffening phenomenon into account. The main goal of this study is to develop a model that predicts the deflection of one-way slabs made of reinforced concrete. The model takes into account concrete contribution in carrying tension between cracks thoroughly considering the stress-strain relationship material model. The model has been developed using MATLAB Program incorporating a numerical technique known as the layered approach. The developed layered approach model was validated using experimental deflection results reported in a previous study. Based on the experimental validation, the model showed its tendency in predicting the real-case deflection of one-way concrete reinforced slabs. It was concluded that different types of tension stiffening models can be added to the program to model deflection and crack propagation.

Keywords: Short-term deflection; Tension stiffening; Modeling; Stress-strain relationship

1 Introduction

Cracking of Reinforced Concrete (RC) member takes place when the tensile stress acting on the flexural mebmer exceeds the tensile strength of the extreme fiber. The diversity in the rigidity of the member throughout its length due to cracking creates difficulty in the estimation of deflection. In a cracked member, the uncracked area has the most rigidity, while it is the smallest in the cracked area (Gilbert & Warner, 1978). However, at intermediate regions between adjacent cracks, due to the bond action, concrete surrounding reinforcement keeps part of its tensile force, which effectively stiffens the member's reaction and decreases deflection. This effect is recognized as tension-stiffening (Gribniak, 2009). If the effect is ignored, Calculated Deflection may be significantly overestimated.

Cracking and tension stiffening (Lam et al., 2010) factors have the greatest impact on the numerical outcomes of concrete members under short-term loading. Numerical techniques are an effective way to examine the performance related to the design and optimization of deflection. They can comprise all conceivable material effects that give reinforced concrete's behavior its complicated nature. Numerical solutions are usually used in finding a solution for structural behavior governed by pre-defined material models. Commonly, a material model consists of equations that illustrate the relation between the stress and strain for a specific material. Material models are classified into different groups and in each group, a detailed principle is governing the stress-strain relation. In each material model, certain parameters are defined, which represent the specification of the characteristics of a specific material. Those parameters could be obtained by performing a nonlinear analysis to experimental results, where curve fittings and statistical methods are used to identify the parameter's values. For example, material models such as stress-strain relationships are implemented to obtain the values of curvature for further analysis of deflection (Sokolov, 2010).

The stress-strain model permits the tensile stresses to be produced using a tensile stress block, in the concrete in the spaces between adjacent fractures. It is commonly more applicable for any variety of shapes in members whether or not subject to axial loads and being bent uniaxially or biaxially. Tensile stress blocks can be efficient when performing full-range analysis under both service & ultimate conditions. The exact tensile stresses' magnitude in a cracked concrete member is incredibly challenging to assess. Therefore, current tensile stress blocks have been produced by suggesting a certain tensile stress block with unspecified parameters. Theoretical moment-curvature or load-deflection curves derived from the experiment are used to obtain the value of these parameters by curve fitting. Tensile stress blocks with linear ascending and descending branches are expressed in terms of $\alpha_1 \ and \ \alpha_2$ and these parameters have a different range of values according to a different type of load, tension reinforcement ratio, applied moment, and effective depth to total depth ratio (Lam *et al.*, 2010).

Recently, numerical techniques have been widely used to simulate structural complexity in real-case scenarios. Main equations are frequently simple and fail to reflect the material's complex multi-factor nature; therefore, don't assure higher statistical precision of deflection for flexural RC members (Sokolov, 2010). One of the numerical techniques that is well-known in calculating the deflection of RC members is the layered approach (Gribniak et al., 2012). A Layer model can be conveniently used for calculating stresses and internal forces at every cross-section along the length of the RC member. For steel reinforcement, stresses can be predicted from the longitudinal strains utilizing its experimental stress-strain curves. Since the applied moment is known, the problem could be solved considering the equilibrium equations for both axial force Eq (1), and bending moment Eq (2).

$$A_o\varepsilon_o - A_1\psi_0 = 0 \quad (1)$$

$$A_1\varepsilon_o - A_2\psi_0 = M \quad (2)$$

$$A_o = \sum_{i=1}^{m} R_{ci}bh_i + E_sA_s + E_sA_s \quad (3)$$

Where, ε_o the strain at the top fiber (reference point), ψ is the curvature, M is the applied moment at the section, and:

$$A_1 = \sum_{i=1}^{m} R_{ci}bh_i\, y_i + E_sA_sd + E_sA_s{}'d' \quad (4)$$

$$A_2 = \sum_{i=1}^{m} R_{ci}bh_i\, y_i{}^2 + E_sA_sd^2 + E_sA_s{}'d'^2 \quad (5)$$

m is the number of concrete layers in each section.

R_{ci} is the material stiffness for each concrete layer.

b is the width of the cross-section.

A_s is the cross-section area of the steel reinforcement in tension.

$A_s{}'$is the cross-section area of the steel reinforcement in compression.

E_s is the elasticity modulus of steel.

d is the depth of reinforcement in tension.

d' is the depth of reinforcement in compression.

y_i is the coordinate of the layer measured downwards from the reference point (top fiber).

h_i is the thickness of the layer.

2 Method of Analysis

The slab was split into several equal sections throughout its entire length. Then, the moment was calculated for each segment. The procedure of calculating short-term deflection using the Stress-Strain relationship layered approach starts by finding the average cracked curvature using the stress-strain material model. However, initially, the values of the curvature and strain (of the top fiber) were assumed to proceed with further calculations. Afterward, the model will substitute those initial values in equations (1) and (2). Unless the two expressions on the left-hand side and the right-hand side of eq (1) & (2) are equaled, MATLAB's solver will assume new initial values of strain and curvature iteratively. The conditions for the averaged crack section used in the MATLAB model script are given below (Carreira DJ & Chu, 1986; Torres et al., 2004).

$$\varepsilon_i \le \varepsilon_{ct}\ , \qquad \mathrm{R_{ci}} = \mathrm{E_C} \tag{6}$$

$$\varepsilon_{ct} \le \varepsilon_i \le \alpha_2\varepsilon_{ct}, \quad \mathrm{R_{ci}} = \mathrm{E_t} \tag{7}$$

$$\alpha_2\varepsilon_{ct} < \varepsilon_i, \qquad \mathrm{R_{ci}} = 0 \tag{8}$$

$$\varepsilon_{ct} = \frac{f_{ctm}}{E_c} \tag{9}$$

$$\varepsilon_i = \varepsilon_o - \psi y_i \tag{10}$$

$$E_t = \frac{\frac{\alpha_1 f_{ctm}(\alpha_2\varepsilon_{ct}-\varepsilon_i)}{(\alpha_2\varepsilon_{ct}-\varepsilon_{ct})}}{\varepsilon_i} \tag{11}$$

ε_i is the strain at the top fiber (reference point).

ε_{ct} is the mean value of the strain of concrete.

ε_i is the strain at the midpoint of each layer.

R_{ci} is the material stiffness for each concrete layer.

E_t is the secant deformation modulus in the descending branch.

α_1,α_2 are dimensionless coefficients considered in this study, the values of α_1 and α_2 are 0.5 and 10, respectively, according to Damjanic & Owen (Lam *et al.*, 2010).

f_{ctm} is the mean concrete tensile strength.

Using the curvature values obtained from the above equations, the deflection values were calculated using the following Equations (9-11) (Eurocode 2, 2008).

$$\theta_i = \theta_{i-1} + 0.5\frac{l}{n}\left(\psi_{m(i)} + \psi_{m(i-1)}\right) \qquad (12)$$

$$\delta_i = \delta_{i-1} + \frac{l}{2}\left(\frac{\theta_{(i)}+\theta_{(i-1)}}{n}\right) \qquad (13)$$

$$\delta_{corrected(i)} = \delta_b \frac{x_i}{l} - \delta_i \qquad (14)$$

Where θ_i, is the cumulative slope for each section, δ_i is the cumulative deflection for each section, and δ_b is the cumulative deflection at the right support.

3 Validation of the Model

To validate the numerical MATLAB model, its results were compared against experimental results which were previously reported in (Abdelatif & Wahab, 2016). In the previous experimental study, the slab was simply supported and tested under a point load positioned at the mid of the beam. In order to conduct an accurate validation of the model, the same dimensions and slab material properties were identical to those reported in the experimental study. The slab material properties and dimensions are shown in Table 1.

Table 1: The Properties of the Slab

Parameter	Value
Length of the slab, L (mm)	2000
Width of the slab, b (mm)	550
Concrete Elastic Modulus, E_c (Gpa)	32.8366
Steel Elastic Modulus, E_s (Gpa)	200
Area of Steel Reinforcement, A_s (mm^2)	1146
Characteristic Cylinder Strength, f_{ck} (Mpa)	30
Mean Characteristic Tensile Strength, f_{ctm}(Mpa)	2.89
Max Pointed load, P (kN)	14.715
Thickness of slab, h (mm)	80

Furthermore, Load-mid span deflection curve obtained from the model was compared to the experimental one as shown in Figure 1. It can clearly be seen that the developed model has a good match with the outcomes of the experiment. Therefore, such models can be used widely to study different flexural members.

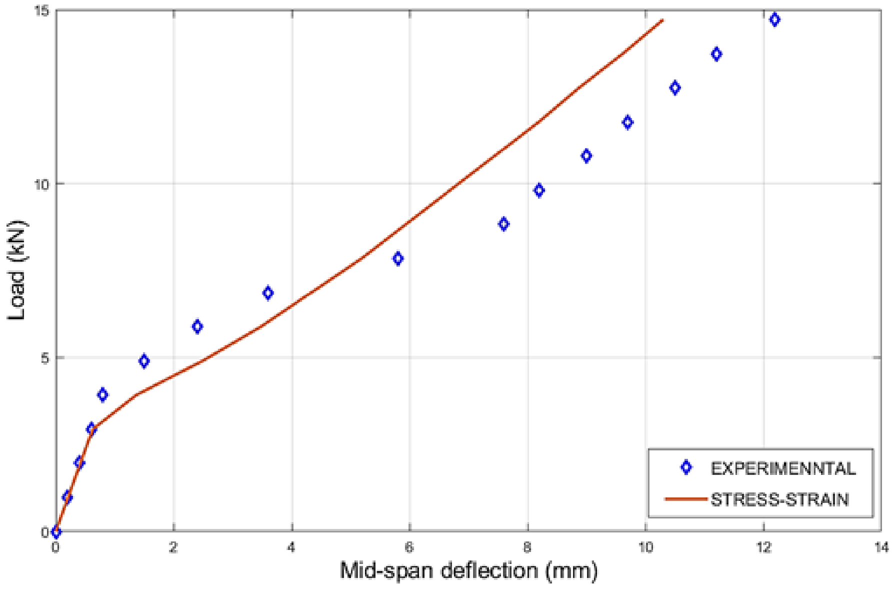

Fig. 1: Load-Deflection for a slab subjected to point load

3.1 Simulation of Strain Contour

In order to visualize the strain results on a concrete surface at different load steps before and after cracking, 2D contours were generated from the results using MATLAB as shown in Figure 2(a). In this example, only in 6 loading steps, as in the experiment, the contours are generated, however it could be generated for uncountable loading steps. As reported in the literature Abdelatif and Wahab (2016), concrete starts cracking when the tensile strain exceeds 0.01 % (i.e 0.0001). Accordingly, this limit is reached at the second loading step where the mid-span load is 3.924 kN in this case. This will directly interpret why the load-deflection curve goes in non-linear behavior after this step. These strains kept increasing as the load increased.

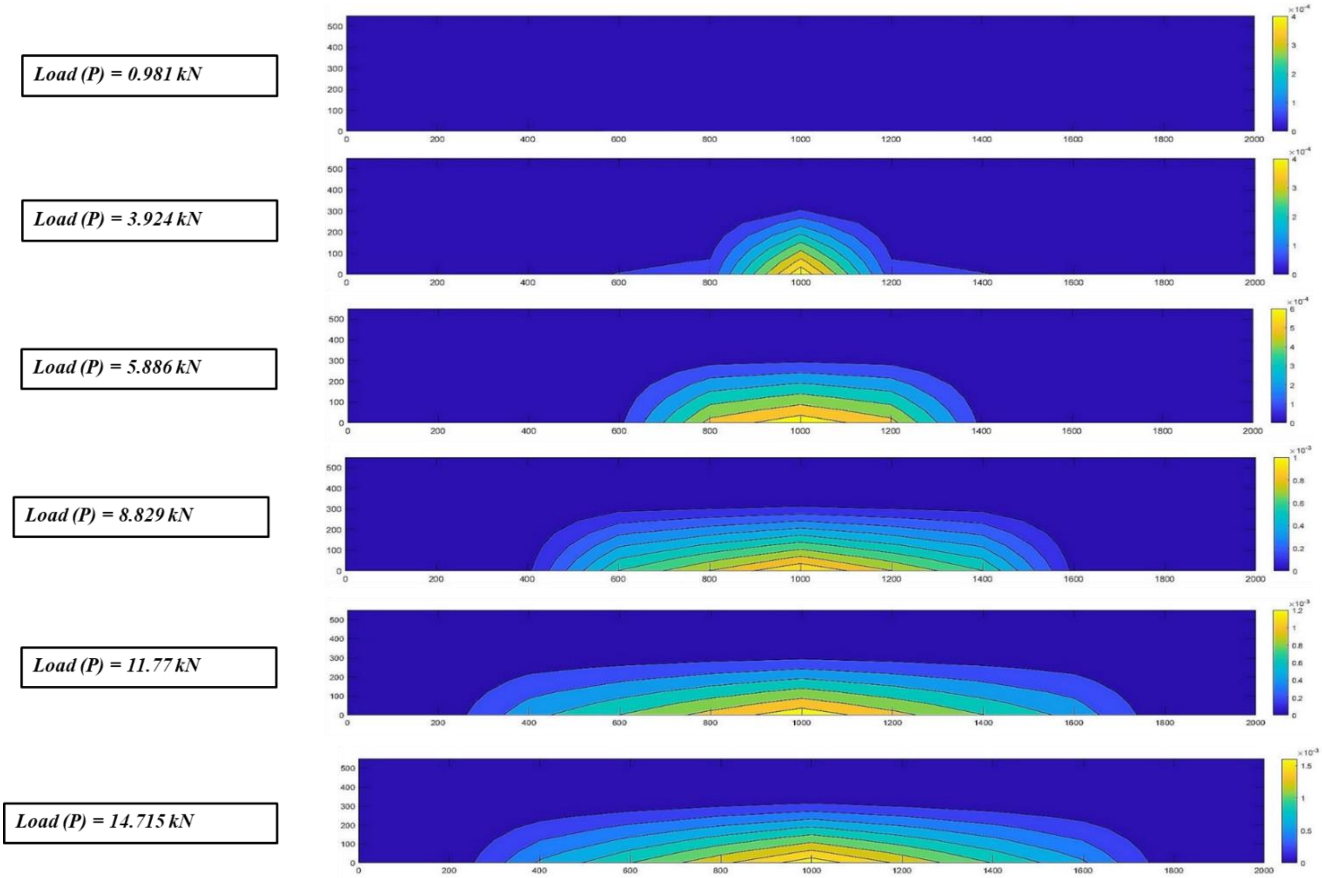

Fig. 2: Simulation of Strain Contour of One-Way Slab

4 Conclusion & Recommendations

In the study, a numerical model is developed to estimate the one-way slabs reinforced concrete's short-term deflection using the layered approach based on the average stress-strain material model. The model has been developed and created from the material model and layered method's fundamental equations using the MATLAB program to solve those equations iteratively. By comparing the developed model deflection results to the experimental deflection results for the same slab, good agreement was observed. One of the advantages of the developed model is that different tension stiffening models can be incorporated to estimate deflection and crack propagation. For future work, such models can be extended to study different reinforced concrete structural members under flexure considering different loading systems and different material models. In connection, the obtained results from this model can be compared to other simulation models that adopt different numerical techniques such as Finite Element Modelling.

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